Book Review

P. D. Panagiotopoulos, *Hemivariational Inequalities: Applications in Mechanics and Engineering*, Springer Verlag, Berlin, New York, 1993.

In 19-th century the term "Applied Mathematics" meant what we call now "Theoretical Mechanics". This theory is based on variational principles first formulated in the 18-th and 19-th centuries. Practical applications of these principles called for the development of the corresponding mathematical theory, and such a theory – Calculus of Variations – became a reality. However, like "la plus belle ne peut donner plus quelle a", Mathematics was unable to give Mechanics more than it had at the time but it had only Smooth Analysis at its disposal. Therefore, as a result, in Mechanics they considered only problems which can be stated and solved with the help of smooth functions and functionals. Prof. P. D. Panagiotopoulos was a pioneer in introducing Nonsmooth Analysis in Mechanics. First he used the J.-J. Moreau and R. T. Rockafellar theory of subdifferentials (for Nonsmooth Convex functions) and applied it to solving nonsmooth problems in Mechanics (see, e.g., Moreau, J.-J., Panagiotopoulos, P. D., and Strang, G. eds: *Topics in Nonsmooth Mechanics*, Birkhauser Verlag, Basel, 1988). Later he used the Clarke subdifferential and Quasidifferential Calculus. The importance of his approach is not only the fact that some very interesting problems were solved so far but, more significantly, his general approach itself represents a breakthrough into a new – nonsmooth – area of Mechanics and into a new type of thinking. The field of problems having variational expressions in inequality form has seen a considerable development in Mathematics, Mechanics, Engineering Sciences and Economics in a very short time. This is mainly due to the fact that new, very efficient, mathematical tools used in this field proved beneficial to the promotion of scientific thought and methodology; open problems have been treated and entirely new categories of interesting problems in Mathematics, Applied Mechanics and several branches of the Engineering Sciences have been mathematically formulated, studied and/or numerically treated. We can distinguish two main directions in the field of inequality problems – that of variational inequalities which has already a research "evolution" of about 30 years and is mainly connected with convex energy functions, and that of hemivariational inequalities which is more "young" – the idea of hemivariational inequalities has a research life of only 10 years – and is connected with nonconvex energy functions. This book deals with problems in Mechanics and Engineering Science whose variational formulations are hemivariational inequalities. The treatment of such problems differs fundamentally from that of inequality problems whose variational forms are variational inequalities, due to the nonconvexity of the energy functions involved. Most of the nonconvex energy functions used are nonsmooth and, thus, the methods of Nonsmooth Analysis are employed for the mathematical study and the numerical treatment of the hemivariational inequalities. The book contains four parts: the "Introductory Topics"(Chapter 1), the "Mechanical Theory" (Chapters 2 to 5), the "Mathematical Theory" (Chapters 6 to 8), and the "Numerical Applications" (Chapters 9 to 15). Part I includes the necessary mathematical background concerning convexity and subdifferential, generalized gradient and duality, elements of the theory of fans and quasidifferentiability. Part II deals with the mechanical aspects of the theory of hemivariational inequalities. In this part the notions of convex and nonconvex superpotentials are defined and, by means of these notions, boundary conditions and material laws expressed through convex and nonconvex superpotentials are introduced. Moreover, the general method for the derivation of hemivariational and variational-hemivariational inequalities is presented as well as a first discussion of the relationship between hemivariational inequalities and substationarity of the potential or complementary energy. Then unloading problems, eigenvalue problems for hemivariational inequalities and the multivalued boundary integral equations which are equivalent to (boundary) hemivariational inequalities are treated. Fuzzy material laws and boundary conditions, and nonconvex dissipation superpotentials are introduced and the corresponding class of generalized standard materials with nonconvex energy functions is studied. Moreover, material laws and boundary conditions expressed by means of fans and quasidifferentials and the corresponding variational expressions are formulated.

Part III deals with the mathematical theory of hemivariational and variational-hemivariational inequalities, as well as their exact relationship to substationarity problems. Moreover, the eigenvalue problem for hemivariational inequalities is studied along with dynamic hemivariational inequalities arising in the theory of von Karman laminated plates and thermoelasticity. The mathematical part of the book concludes with the formulation and study of the optimal control problem of systems governed by hemivariational inequalities. In this part of the book, where existence and approximation results for the solution(s) of hemivariational inequalities are proved, the mathematical results are rigorously derived by using the tools of Nonsmooth Analysis and Nonlinear Functional Analysis.

Part IV is devoted to the numerical applications and takes the largest part of the book. Numerical applications related to real engineering problems are given. Most of the problems treated cannot be accurately solved by other, more classical, numerical techniques due to the strong nonlinearities arising from nonmonotone and multivalued stress-strain or reaction-displacement behaviour. This part of the book describes innovative numerical techniques and points to concrete engineering applications. The last Chapter deals with hemivariational inequalities defined on fractal geometries and an attempt is made to adapt the numerical techniques for hemivariational inequalities to a neurocomputing environment.

The book contains highly innovative material and original results of nonconvex energy theories both from the mathematical and the numerical points of view.

University of St-Petersburg, Russia V. F. DEMYANOV